Application of a New Method in Adaptive Control

Lee Gregor Hofmann*
Systems Technology Inc., Princeton, N. J.

The adaptive control function technique is applied to design a remarkably simple adaptive lateral stability augmentation system for a hypothetical manned, lifting-body entry vehicle. This system is proven stable under certain ideal conditions. The general principles of the adaptive control function technique are summarized and used to motivate the steps in the design for this specific multicontrol-point application. The basic operating principle is that sums of properly modulated conventional linear feedforward and feedback signals provide the control functions needed to obtain specified responses in a number of output variables of a linear controlled element. System performance is demonstrated via computer simulation. Data show the adaptive system highly capable of coping with the unstable nature of the vehicle as well as those nonideal effects (time-varying vehicle-vehicle model mismatch, disturbance inputs, outer control loops, and noise suppression filtering) not considered in the analytical design. A comparison of the gains set by the adaptive system, with those selected by a competent aircraft flight control system analyst as optimum, confirms in still another way the reasonable performance of this system.

Introduction

RECENT research has led to a new method in adaptive control and application of this new method to a significant flight control problem. The objective was to produce an adaptive system that is 1) simple to mechanize, 2) amenable to mathematical analysis, and 3) proven to be stable. In addition, the new method should be a generally applicable technique in distinction to the development of a specific design. No adaptive method reported in the existing literature of this field satisfies all four of the above requirements. The adaptive control function technique does satisfy these requirements and is therefore a noteworthy contribution to the art.

The adaptive control function technique described herein is based upon Lion's work on the stability and rapid convergence of parameter tracking systems.[1] The key innovations are the use of an error, which is a vector quantity, and the requirement that each component of the error vector have some algebraic dependency upon the adaptive gains. The adaptive control function technique also has a somewhat less strong connection with the model reference adaptive technique set forth by Osburn, Whitaker, and Kezer in Ref. 2. The main link here is that conceptually, at least, both techniques use a reference model and weighting filters. The criteria being minimized and the adaptive system mechanizations, however, are quite different for the two techniques.

The material in this paper pertains directly to the design of an adaptive lateral stability augmentation system (SAS) for a hypothetical manned, lifting-body, entry vehicle (MLEV) that uses this new method. By developing the theory within the context of an application, the reader will be spared from considering many relatively unimportant details of the general theory. The general theory appears in Appendix B of Ref. 3, and is summarized below.

Summary of the Adaptive Control Function Technique

The controlled element is assumed to be constant coefficient and linear. Its transfer functions, which are assumed to be proper rational functions of s, are given by

$$cu = (C/\Delta)fu + (D/\Delta)du \tag{1}$$

$$mu = (M/\Delta)du + (N/\Delta)du \tag{2}$$

C, D, M, and N are matrices of transfer function numerators. Δ is the characteristic polynomial for the controlled element. The u's are conformable vectors with unity elements. The c, m, f, and d are diagonal matrices of primary feedback variables (variables to be controlled), secondary feedback variables, control variables and disturbance variables, respectively. It follows that fu, for example, is a vector of control variables, etc., f and c are equal dimension.

The control law is

$$fu = [r - c \mid -Fm]\mathcal{L}^{-1}k \tag{3}$$†

where r is a diagonal matrix of commands, k is a vector of adaptively adjusted gains and the matrix F contains the feedback transfer functions. Fixed gain control paths are assumed to be included in Eqs. (1) and (2). A model of the closed-loop system is implicit in the adaptive control function technique, but is not explicitly mechanized. This implicit model is taken to have the same form as the closed-loop system given by Eqs. (1–3). Therefore, those equations apply to the model as well when m subscripts are added. The only other requirement on the model is that R be a positive definite matrix with finite elements where

$$R = (\Delta_m C_m^{-1} C/\Delta)_{s=\infty} \tag{4}$$

A vector of error signals, given by

$$e = [r - c \mid -Fm]\mathcal{L}^{-1}k_m - \Delta_m C_m^{-1} cu \tag{5}$$

is used in the differential equation governing adaptive gain adjustment

$$\dot{k} = A[r - c \mid -Fm]'\mathcal{L}^{-1}e \tag{6}$$

A is a diagonal matrix with positive elements. The elements are the adaptive loop gains. The filter transfer function matrix, $\Delta_m C_m^{-1}$, required for mechanizing Eq. (5) is the inverse of the controlled element model transfer function matrix. Coupling numerator identities of the multiloop analysis technique[4] are then used to reduce $\Delta_m C_m^{-1}$ to its most elementary form and to obtain the equations for the single multiport filter having the required transfer functions.

The remainder of this section is devoted to describing the most appropriate SAS for improving the handling qualities

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* Principal Specialist. Member AIAA.

† $\mathcal{L}^{-1}$ denotes inverse Laplace transform of quantity in brackets.

for the MLEV. Following sections, in turn, consider analytical development of the adaptive control function system for this application, simulation results, and conclusions.

Controlled Element and Feedback Quantities

The lateral MLEV dynamics are characterized by transfer functions for the theoretical portion of this study. These are obtained in the usual way from the linearized Laplace transformed equations of motion in body-fixed axes (Appendix A, Ref. 5).

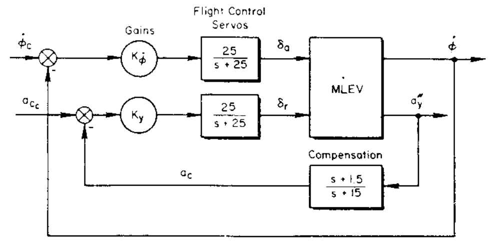

Fig. 1 Block diagram of MLEV lateral SAS.

$$\begin{bmatrix} (s - Y_v) & -[(sW_0 + g\cos\theta_0)/V_{T_0}] & (sU_0 - g\sin\theta_0)/sV_{T_0} & 0 & 0 \\ -L_\beta' & s(s - L_p') & -L_r' & 0 & 0 \\ -N_\beta' & -sN_p' & (s - N_r') & 0 & 0 \\ 0 & s & \tan\theta_0 & -1 & 0 \\ -V_{T_0}Y_v & sj^2l_z & -sl_x & 0 & 1 \end{bmatrix} \begin{Bmatrix} \beta \\ p/s \\ r \\ \dot{\varphi} \\ a_y'' \end{Bmatrix} = \begin{bmatrix} 0 & Y_{\delta_r}/V_{T_0} \\ L_{\delta_a}' & L_{\delta_r}' \\ N_{\delta_a}' & N_{\delta_r}' \\ 0 & 0 \\ 0 & y_{\delta_r} \end{bmatrix} \begin{Bmatrix} \delta_a \\ \delta_r \end{Bmatrix} \qquad (7)$$

The transfer functions are expressed in terms of ratios of numerator polynomials $N^{(\;)}_{(\;)}$ to the characteristic polynomial Δ. Thus, for example, the unaugmented aileron to roll angle rate transfer function is $N_{\delta_a}{}^{\dot{\varphi}}/\Delta$. The subscript on N indicates the input for the transfer function, whereas the superscript indicates the output.

The problem at hand is a multicontrol-point problem. Because of this, it is convenient to use the so-called coupling numerators of the multiloop analysis technique developed in Ref. 4. The coupling numerator is convenient because it expresses in a simplified and compact way a combination of numerators and the characteristic polynomial that occurs frequently in multicontrol-point problems. Here the aileron-to-roll rate, rudder-to-lateral acceleration coupling numerator $N_{\delta_a\delta_r}{}^{\dot{\varphi}a_y''}$ is needed.

$$N_{\delta_a\delta_r}{}^{\dot{\varphi}a_y''} = (N_{\delta_a}{}^{\dot{\varphi}}N_{\delta_r}{}^{a_y''} - N_{\delta_r}{}^{\dot{\varphi}}N_{\delta_a}{}^{a_y''}/\Delta) \qquad (8)$$

The characteristic polynomial Δ is always an exact factor of the numerator on the RHS of Eq. (8). This suggests that easier means than direct evaluation of Eq. (8) can be found to calculate $N_{\delta_a\delta_r}{}^{\dot{\varphi}a_y''}$. Indeed, this is the case. The coupling numerator can be calculated by a method analogous to Cramer's rule. That is, $N_{\delta_a\delta_r}{}^{\dot{\varphi}a_y''}$ can be obtained from the Laplace transformed aircraft equations of motion by substituting the δ_a control effectiveness column into the $\dot{\varphi}$ column of the characteristic matrix and the δ_r control effectiveness column into the a_y'' column of the characteristic matrix and then computing the determinant of the result. The coupling numerator notation is suggestive of this operation.

The feedbacks selected in the competing systems surveys of Ref. 3 as most appropriate for improving the handling qualities are 1) lateral acceleration, a_y'', measured l_x = 3.38 ft forward of the vehicle center of gravity compensated by a lead/lag network $(s + 1.5)/(s + 15)$, and fed to rudder; and 2) roll rate feedback to aileron (derived roll rate obtained from vertical gyro operating as a free gyro).

Figure 1 is a block diagram of this system. Clearly, the variables to be controlled are $\dot{\varphi}$ and a_c. The MLEV equations can be expressed in matrix form and in terms of transfer functions as

$$\begin{Bmatrix} \dot{\varphi} \\ a_y'' \end{Bmatrix} = \frac{1}{\Delta}\begin{bmatrix} N_{\delta_a}{}^{\dot{\varphi}} & N_{\delta_r}{}^{\dot{\varphi}} \\ N_{\delta_a}{}^{a_y''} & N_{\delta_r}{}^{a_y''} \end{bmatrix}\begin{Bmatrix} \delta_a \\ \delta_r \end{Bmatrix} \qquad (9)$$

and the control law, neglecting the flight control servo dynamics, as

$$\begin{Bmatrix} \delta_a \\ \delta_r \end{Bmatrix} = \begin{bmatrix} K_{\dot{\varphi}} & 0 \\ 0 & K_y \end{bmatrix}\left(\begin{Bmatrix} \dot{\varphi}_c \\ a_{c_c} \end{Bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & (s + 1.5)/(s + 15.0) \end{bmatrix}\begin{Bmatrix} \dot{\varphi} \\ a_y'' \end{Bmatrix}\right) \qquad (10)$$

The system surveys also show that reasonable choices for adjustable gains are $K_{\dot{\varphi}}$ and K_y. $K_{\dot{\varphi}}$ should be adjustable to compensate for the large magnitude changes in the control effectiveness derivatives. K_y should be adjustable because there is no constant value of this gain that would not produce instability at some flight conditions. On the other hand, there is no alternative to lateral acceleration feedback to rudder capable of coping with the roll reversal problem at flight condition‡ 810 and the ω_φ/ω_d problem at 850 and capable of supplying some modest amount of dutch roll damping. In particular, yaw rate feedback to rudder cannot accomplish all these objectives simultaneously. The optimum controller gains determined in the systems surveys are given in Table 1.

Although it is the case that these gains are based on calculations that assume the SAS consists of linear, constant coefficient feedbacks of motion quantities, this assumption of the basic system form is best for this application for reasons of flight safety. That is, the closed-loop system should be capable of delivering at least marginal performance with the adaptive system turned off, and SAS gains set manually to suitable values for emergency operation. Manual settings would, of course, have to be changed one or possibly two times during the flight as Mach number changes in order to maintain a stability margin.

Analysis

Once the SAS feedback quantities have been selected, the next steps are to choose a model for the closed-loop system, define the errors for adaptive system operation, choose an adaptive gain adjustment law, and then examine the stability of the adaptive gain adjustments under certain ideal circumstances. The final, and perhaps most important step of all, is that of making modifications to the resulting theoretical design to accommodate the nonideal effects that are a part of any system with real components. The practical modification techniques presented are for coping with the lags inherent in the flight control servos, replacing pure differentiations with pseudo-differentiators and for adding additional low-pass filtering to combat noise.

Table 1 Optimum controller gains

Flt. Cond., sec	$-K_y\vert_{dB}$	$-K_{\dot{\varphi}}\vert_{dB}$
725	NA^a	7.0
810	−10	0.0
840	−20	−10.0
850	−20	−10.0
865	−10	0

[a] Rudder control effectiveness is zero.

‡ Flight conditions are specified by elapsed time in seconds from an arbitrary reference point along a nominal trajectory.

Closed-Loop System Model

An implicit model of the closed-loop system is used in the adaptive control function scheme. This model has the same configuration as the system in Fig. 1. What is more, the transfer functions for the MLEV model are approximately those for the actual MLEV at one flight condition. Since the system feedbacks have been chosen because they can provide a good SAS at any flight condition, this model of the closed-loop system will surely be a good one. The settings for the SAS model gains are, of course, those calculated in the systems surveys for that particular flight condition. Another reason for choosing the model to approximate closely the system at one flight condition is the intuitive expectation that this will tend to require smaller control surface deflections than a model less closely related to the characteristics of the actual system. Equations (8–10) apply as well to the model if it is understood that the subscript m is added to designate model values. The model of the closed-loop system itself is not used explicitly in adaptive control function systems. Instead, part of the mathematical inverse of this model is used. The result is another set of equations for generating model values of the command inputs, $\dot{\varphi}_{c_m}$ and $a_{c_{c_m}}$, from the $\dot{\varphi}$ and a_y'' signals. Adding m subscripts to denote "model" to Eq. (9), computing the inverse, and simplifying the result using Eq. (8) gives

$$\begin{Bmatrix} \delta_a \\ \delta_r \end{Bmatrix}_m = \frac{1}{(N_{\delta_a\delta_r}{}^{\dot{\varphi}a_y''})_m} \begin{bmatrix} N_{\delta_r}{}^{a_y''} & -N_{\delta_r}{}^{\dot{\varphi}} \\ -N_{\delta_a}{}^{a_y''} & N_{\delta_a}{}^{\dot{\varphi}} \end{bmatrix}_m \begin{Bmatrix} \dot{\varphi} \\ a_y'' \end{Bmatrix} \quad (11)$$

Adding m subscripts to Eq. (10), eliminating the dependence upon δ_{a_m} and δ_{r_m} by using Eq. (11), and solving for $\dot{\varphi}_{c_m}$ and $a_{c_{c_m}}$ gives

$$\begin{Bmatrix} \dot{\varphi}_c \\ a_{c_c} \end{Bmatrix}_m = \left(\begin{bmatrix} 1 & 0 \\ 0 & (s+1.5)/(s+15.0) \end{bmatrix} + \frac{1}{(N_{\delta_a\delta_r}{}^{\dot{\varphi}a_y''})_m} \times \begin{bmatrix} 1/K_{\dot{\varphi}} & 0 \\ 0 & 1/K_y \end{bmatrix}_m \begin{bmatrix} N_{\delta_r}{}^{a_y''} & -N_{\delta_r}{}^{\dot{\varphi}} \\ -N_{\delta_a}{}^{a_y''} & N_{\delta_a}{}^{\dot{\varphi}} \end{bmatrix}_m \right) \begin{Bmatrix} \dot{\varphi} \\ a_y'' \end{Bmatrix} \quad (12)$$

Choosing Appropriate Errors

The key property of appropriate errors is that they must contain some algebraic dependency upon each adaptive gain in addition to whatever other functional dependency might exist. There also must be a number of independent errors equal to the number of control functions that depend algebraically upon the adaptive gains. For the present case, there are two such control functions, δ_a and δ_r, and two errors are required.

The error signals are defined to be proportional to the differences between the actual commands and the model representations of the commands.

$$e_a = (\dot{\varphi}_c - \dot{\varphi}_{c_m})K_{\dot{\varphi}_m}$$
$$e_r = (a_{c_c} - a_{c_{c_m}})K_{y_m} \quad (13)$$

The signal e_a represents error in the aileron control channel: the subscript a standing for aileron. The signal e_r represents error in the rudder control channel: the subscript r standing for rudder. More convenient expressions for the errors are in terms of the system servo errors, $(\dot{\varphi}_c - \dot{\varphi})$ and $(a_{c_c} - a_c)$, and the model control deflections, δ_{a_m} and δ_{r_m}.

$$e_a = (\dot{\varphi}_c - \dot{\varphi})K_{\dot{\varphi}_m} - \delta_{a_m}$$
$$e_r = (a_{c_c} - a_c)K_{y_m} - \delta_{r_m} \quad (14)$$

These equations are the mechanizational basis for the system and will be needed later. To show the basic algebraic dependence of these errors upon $K_{\dot{\varphi}}$ and K_y, subtract the control function expressions for the system (refer to Fig. 1, and neglect the flight control servo dynamics),

$$0 = K_{\dot{\varphi}}(\dot{\varphi}_c - \dot{\varphi}) - \delta_a$$
$$0 = K_y(a_{c_c} - a_c) - \delta_r \quad (15)$$

from the respective error equations [Eq. (14)],

$$e_a = (\dot{\varphi}_c - \dot{\varphi})(K_{\dot{\varphi}_m} - K_{\dot{\varphi}}) - (\delta_{a_m} - \delta_a)$$
$$e_r = (\dot{a}_{c_c} - a_c)(K_{y_m} - K_y) - (\delta_{r_m} - \delta_r) \quad (16)$$

Obviously, the errors display the required algebraic dependence upon the adaptive gains. This is so even when $(\delta_{a_m} - \delta_a)$ and $(\delta_{r_m} - \delta_r)$ are identically zero, as long as the controlled variables $\dot{\varphi}$ and a_c are not equal to their commanded values $\dot{\varphi}_c$ and a_{c_c}.

Stability Characteristics and Gain Adjustment Law

Stability is always vitally important. A crucial objective is to obtain a system that is stable under reasonably defined ideal conditions. This can be accomplished. Notice that from the synthesis point of view, system stability rather than satisfaction of some criterion function is the prime objective. After a stable design is obtained, the criterion function implied can be computed as a matter of curiosity. Elaboration upon "reasonably defined ideal conditions" will be deferred until the equations are set down.

Rewrite Eq. (16) in terms of the gain differences, $\Delta K_{\dot{\varphi}}$ and ΔK_y, for compactness

$$e_a = (\dot{\varphi}_c - \dot{\varphi})\Delta K_{\dot{\varphi}} - (\delta_{a_m} - \delta_a)$$
$$e_r = (a_{c_c} - a_c)\Delta K_y - (\delta_{r_m} - \delta_r) \quad (17)$$

where the gain differences are defined by

$$\Delta K_{\dot{\varphi}} = K_{\dot{\varphi}_m} - K_{\dot{\varphi}}$$
$$\Delta K_y = K_{y_m} - K_y \quad (18)$$

The next step is to choose the rules by which gain changes will be made. The gain adjustment laws are defined as

$$\dot{K}_{\dot{\varphi}} = A_{\dot{\varphi}}(\dot{\varphi}_c - \dot{\varphi})e_a$$
$$\dot{K}_y = A_y(a_{c_c} - a_c)e_r \quad (19)$$

where $A_{\dot{\varphi}}$ and A_y are positive constants. They are the adaptive loop gains. In terms of the gain-differences defined by Eq. (18), the adjustment laws are

$$\Delta\dot{K}_{\dot{\varphi}} = -A_{\dot{\varphi}}(\dot{\varphi}_c - \dot{\varphi})^2\Delta K_{\dot{\varphi}} + f_{\dot{\varphi}}$$
$$\Delta\dot{K}_y = -A_y(a_{c_c} - a_c)^2\Delta K_y + f_y \quad (20)$$

where

$$f_{\dot{\varphi}} = A_{\dot{\varphi}}(\dot{\varphi}_c - \dot{\varphi})(\delta_{a_m} - \delta_a)$$
$$f_y = A_y(a_{c_c} - a_c)(\delta_{r_m} - \delta_r) \quad (21)$$

since $K_{\dot{\varphi}_m}$ and K_{y_m} are constants.

Equations (20) are first-order differential equations with time-varying gains and forcing functions given by Eq. (21). They describe the dynamic response of the adaptive loops. Equations (20) indicate that the adaptive system is stable in the sense that for $f_{\dot{\varphi}}$ and f_y equal to zero, $|\Delta K_{\dot{\varphi}}|$ and $|\Delta K_y|$ are monotonically decreasing time functions. This follows from the fact that the time-varying gains in the homogeneous equations are always nonpositive. That is, for example in the $\Delta K_{\dot{\varphi}}$ equation, the term $-A_{\dot{\varphi}}(\dot{\varphi}_c - \dot{\varphi})^2$ is either a negative number or, at worst, zero. Thus, when $f_{\dot{\varphi}}$ and f_y are identically zero, $\Delta K_{\dot{\varphi}}$ and ΔK_y cannot diverge. In addition to these favorable stability features, there is another important attribute of the system. The adjustable gains are uncoupled. An offset in one gain will not result in a transient disturbance of the other gain when $f_{\dot{\varphi}}$ and f_y are zero. Traditional multiparameter adaptive systems have often suffered from coupling which, with increasing adaptive loop gain, generally tends to cause oscillatory behavior.

Now, what conditions are implied by $f_{\dot{\varphi}}$ and f_y equal to zero? These conditions will in fact be those "reasonably defined ideal conditions" mentioned at the beginning of the subsection. Hence, the conditions are actually requirements for validity of the conclusions on stability. Notice that $f_{\dot\varphi}$ and f_y are zero over all time if, and only if $(\dot\varphi_c - \dot\varphi)$ or $(\delta_{a_m} - \delta_a)$ and $(a_{c_c} - a_c)$ or $(\delta_{r_m} - \delta_r)$ are zero over all time. When a servo error, $(\dot\varphi_c - \dot\varphi)$ and/or $(a_{cc} - a_c)$, is zero, the following observations hold: 1) The system output quantity related to the servo error, which is zero, is exactly equal to the commanded quantity; and 2) When a servo error is zero, the rate of change of the corresponding adaptive gain is zero. [See Eq. (19)]. The first observation leads to the conclusion that if a servo error is zero for all time, there is no need for adaptive action in that channel. The second observation leads to the conclusion that if a servo error is zero for all time, indeed, there will be no adaptive action in that channel, a trivial case. The alternative way for $f_{\dot\varphi}$ or f_y to be zero is for $(\delta_{a_m} - \delta_a)$ or $(\delta_{r_m} - \delta_r)$, respectively, to be zero. If $f_{\dot\varphi}$ and f_y are to be zero over all time in the nontrivial case $(\delta_{a_m} - \delta_a)$ and $(\delta_{r_m} - \delta_r)$ must be zero over all time except at isolated instants when a servo error is zero. Conditions for $(\delta_{a_m} - \delta_a)$ and $(\delta_{r_m} - \delta_r)$ to be zero for all time are

1) No vehicle-vehicle model mismatch (stated another way, the vehicle and the vehicle model must have precisely the same transfer functions).

2) The initial conditions on the corresponding vehicle and vehicle model variables must be the same.

3) No disturbance inputs may act on the vehicle.

Although these restrictive assumptions are severe, the stability for this special case of Eq. (20) is a most gratifying and important characteristic. This special case provides a firm and attractive theoretical basis for adaptive system operation.

Next, consider the stability of the errors, e_a and e_r, under the same restrictive assumptions. Equations (17) become

$$\begin{aligned} e_a &= (\dot\varphi_c - \dot\varphi)\Delta K_{\dot\varphi} \\ e_r &= (a_{c_c} - a_c)\Delta K_y \end{aligned} \tag{22}$$

Clearly these errors are always asymptotically stable to zero since either 1) $\Delta K_{(\ )}$ goes to zero by virtue of Eq. (20), or 2) $\Delta K_{(\ )}$ goes to a constant value by virtue of Eq. (20). This implies that the servo error is zero since otherwise $\Delta K_{(\ )}$ would approach zero. Under these assumptions, if the gain adjustment law given by Eq. (19) is a steepest descent law, the criterion implied (to within an arbitrary, positive multiplicative constant here taken as unity) is

$$J = 1/2(A_{\dot\varphi}e_a^2 + A_y e_r^2) \tag{23}$$

That this is so can be verified by computing $\partial J/\partial\Delta K_{\dot\varphi}$ and $\partial J/\partial \Delta K_y$ using Eq. (22) and noting that

$$\begin{aligned} \Delta\dot K_{\dot\varphi} &= -(\partial J/\partial\Delta K_{\dot\varphi}) = -A_{\dot\varphi}(\dot\varphi_c - \dot\varphi)e_a \\ \Delta\dot K_y &= -(\partial J/\partial\Delta K_y) = -A_y(a_{c_c} - a_c)e_r \end{aligned} \tag{24}$$

Certain observations can be made about stability for the more general case of Eq. (20) when the forcing terms are not zero. First of all, when the MLEV model is a reasonably close approximation to the actual MLEV vehicle, the forcing terms will be small in the absence of disturbance inputs. This conclusion follows from the fact that each forcing term contains a factor $\delta_{(\ )m} - \delta_{(\ )}$ that becomes vanishingly small as the MLEV model and the actual MLEV become equivalent. Nevertheless, there actually will be some small amount of forcing due to this mismatch, and the adjusting gains will accordingly be perturbed. As a matter of fact, this same effect also will cause a small amount of cross-coupling between gains through the forcing terms. Neither of these nonideal effects is expected to be troublesome. However, the question of stability of the $\Delta K_{(\ )}$ responses remains open for the most general case.

Mechanizing the System

With the theoretical development of the adaptive system accomplished, the next task is to identify ways for realizing the system within the limitations of physical devices. The mechanizational basis for the adaptive system is given by the equations below:

$$\begin{aligned} e_a^* &= (\dot\varphi_c - \dot\varphi)^* K_{\dot\varphi_m} - \delta_{a_m}{}^* = F(s)e_a \\ e_r^* &= (a_{c_c} - a_c)^* K_{y_m} - \delta_{r_m}{}^* = F(s)e_r \end{aligned} \tag{25}$$

$$\begin{aligned} \dot K_{\dot\varphi} &= A_{\dot\varphi}(\dot\varphi_c - \dot\varphi)^* e_a^* \\ \dot K_y &= A_y(a_{c_c} - a_c)^* e_r^* \end{aligned} \tag{26}$$

These equations are quite similar to Eqs. (14) and (19). The exception is that starred quantities are used in Eqs. (25) and (26). The starred quantities are obtained by passing the corresponding unstarred quantities through high-frequency cut-off filters with transfer functions, $F(s)$. Subsequent discussion will make the need for this filtering in practical mechanization apparent.

Examination of Eqs. (25) and (26) shows that aside from the signals available in the system $(\dot\varphi_c - \dot\varphi)$ and $(a_{c_c} - a_c)$, δ_{a_m} and δ_{r_m} also will be required. These are generated by filtering the outputs of the vehicle sensors, $\dot\varphi$ and a_y'' with transfer functions representing the mathematical inverse of the MLEV model. Refer to Eq. (11). The problem here is to realize these transfer functions. This may be accomplished with a single filter. Consider the vehicle equations of motion, Eq. (7), as they pertain to the MLEV model. Certain approximations can be made in the model for the sake of simplicity. Namely, θ_0, $(l_x + Y_{\delta_r}/N_{\delta_r}')$, l_z, L_r', L_p', N_p' and N_r' can be set to zero. Y_{δ_r} can be set to zero in the sideslip equation but not in the lateral acceleration equation.§ The term $(g/V_{T_0})\cos\theta_0\varphi$ is set to zero in the sideslip equation. The effects of all these approximations are negligible. In fact, if similar approximations are made in the vehicle equations, the short-term time responses are imperceptably changed. This is also true for the long-term responses with the exception of $(g/V_{T_0})\cos\theta_0\varphi$ approximation effect. All these considerations can be combined to write the equations of motion for the MLEV model as

$$\begin{bmatrix} (s - Y_v) & -W_0/V_{T0} & U_0/V_{T_0} & 0 \\ -L_\beta' & s & 0 & 0 \\ -N_\beta' & 0 & s & 0 \\ -V_{T_0}Y_v & 0 & sY_{\delta_r}/N_{\delta_r}' & 1 \end{bmatrix}_m \begin{Bmatrix} \beta \\ \dot\varphi \\ r \\ a_y'' \end{Bmatrix} = \begin{bmatrix} 0 & 0 \\ L_{\delta_a}' & L_{\delta_r}' \\ N_{\delta_a}' & N_{\delta_r}' \\ 0 & Y_{\delta_r} \end{bmatrix}_m \begin{Bmatrix} \delta_a \\ \delta_r \end{Bmatrix}_m \tag{27}$$

It is possible to write down a set of equations having the same transfer functions as Eq. (11) by inspection of Eq. (27). To do this, merely rearrange the columns of the matrices in Eq. (27) so the determinants that express $N_{\delta_a}{}^{\dot\varphi}$, $N_{\delta_a\delta_r}{}^{\dot\varphi a_y''}$ etc., in Eq. (11) form the transfer functions between the variables indicated in Eq. (11). This "inspection" amounts to applying Cramer's rule in reverse. The result is Eq. (28).

$$\begin{bmatrix} (s - Y_v) & 0 & U_0/V_{T_0} & 0 \\ -L_\beta' & L_{\delta_a}' & 0 & L_{\delta_r}' \\ -N_\beta' & N_{\delta_a}' & s & N_{\delta_r}' \\ -V_{T_0}Y_v & 0 & sY_{\delta_r}/N_{\delta_r}' & Y_{\delta_r} \end{bmatrix}_m \begin{Bmatrix} x_1 \\ \delta_a \\ x_3 \\ \delta_r \end{Bmatrix}_m = \begin{bmatrix} -W_0/V_{T_0} & 0 \\ s & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}_m \begin{Bmatrix} \dot\varphi \\ a_y'' \end{Bmatrix} \tag{28}$$

§ Y_v also could be set to zero insofar as the accuracy of the approximations is concerned. It is retained here so that the factors of the characteristic polynomial of Eq. (11) $(N_{\delta_a\delta_r}{}^{\dot\varphi a_y''})_m$ will have a positive damping ratio.

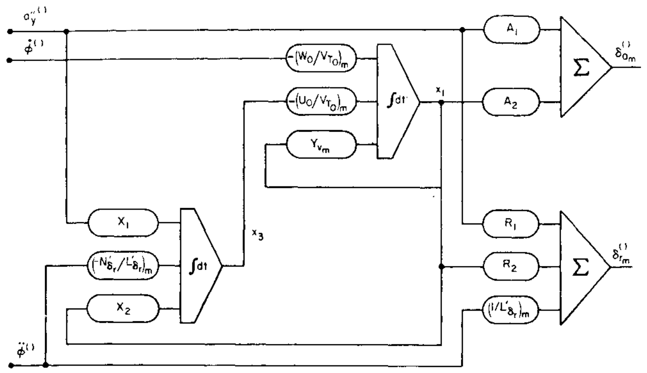

Fig. 2 Mechanization of filter.

This filter is only second order and may be mechanized quite simply using operational elements. The mechanization resulting after the algebraic loops in Eq. (28) are eliminated is shown in Fig. 2. The values of the filter coefficients are given in Table 2. Notice the implied requirement in Fig. 2 to either measure $\ddot{\varphi}$ or calculate $\ddot{\varphi}$ by differentiating $\dot{\varphi}$. Neither alternative is really acceptable. It is possible to get around this point by using a pseudo-differentiator to obtain $\ddot{\varphi}^*$ from $\dot{\varphi}$ and by inserting a cut-off filter in all the other input paths to compensate for the cut-off characteristics of the pseudo-differentiator. This amounts to specifying part of the filter $F(s)$ that appears in Eq. (25). That is, if the pseudo-differentiator has a transfer function $(s/T)/(s + 1/T)$, then $F(s)$ will contain the factor $(1/T)/(s + 1/T)$. To this point, the flight control servo dynamics have been neglected. It is clear that the servo dynamics will interpose between the gains, $K_{\dot{\varphi}}$ and K_y, and the control surface deflections δ_a and δ_r, respectively. Refer to Fig. 1. Flight control servo dynamics can be included by noting that the flight control servos act in the same manner as factors of $F(s)$ operating upon $\dot{\varphi}$ and a_y''. By merely placing similar factors in the $F(s)$ operating upon $(\dot{\varphi}_c - \dot{\varphi})$ and $(a_{c_c} - a_c)$, the relationship of Eq. (25) can be preserved. Assuming that both flight control servos have the same transfer function, $25/(s + 25)$ and taking $1/T$ in the pseudo-differentiator to be 15/sec, the following equations will hold:

$$(\dot{\varphi}_c - \dot{\varphi})^* = [15/(s + 15)][25/(s + 25)](\dot{\varphi}_c - \dot{\varphi}) \quad (29)$$

$$(a_{c_c} - a_c)^* = [15/(s + 15][25/(s + 25)](a_{c_c} - a_c) \quad (30)$$

$$\ddot{\varphi}^* = [15s/(s + 15)]\dot{\varphi} \quad (31)$$

$$\dot{\varphi}^* = \{[15/(s + 15)]\dot{\varphi}\} \quad (32)$$

$$a_y''^* = \{[15/(s + 15)]a_y''\} \quad (33)$$

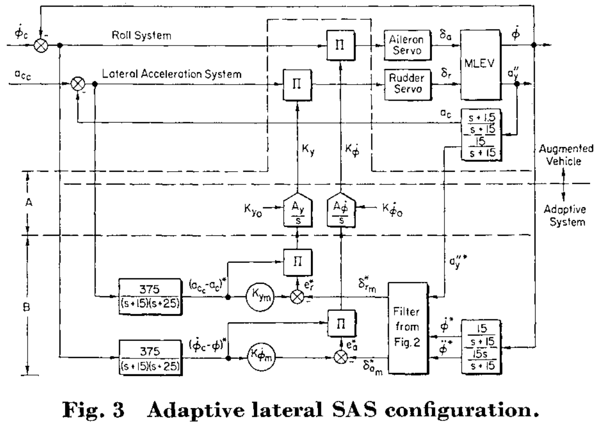

Fig. 3 Adaptive lateral SAS configuration.

Table 2 Filter coefficients

Coefficient	Literal Expression[a]
A_1	$[-N_{\delta_r}'/(N_{\delta_a}'Y_{\delta_r})]_m$
A_2	$[N_{\delta_r}'(N_\beta'/N_{\delta_r}' - Y_v/V_{\delta_r}^*)/N_{\delta_a}']_m$
R_1	$[N_{\delta_r}'L_{\delta_a}'/(Y_{\delta_r}N_{\delta_a}'L_{\delta_r}')]_m$
R_2	$[L_\beta'/L_{\delta_r}' - N_\beta'L_{\delta_a}'/(U_{\delta_a}'L_{\delta_r}') + Y_vN_{\delta_r}'L_{\delta_a}'/(Y_{\delta_r}^*N_{\delta_a}'L_{\delta_r}')]_m$
X_1	$[-N_{\delta_r}'L_{\delta_a}'(N_{\delta_r}' - N_{\delta_a}'L_{\delta_r}'/L_{\delta_a}')/(Y_{\delta_r}N_{\delta_a}'L_{\delta_r}')]_m$
X_2	$[N_{\delta_r}'\{L_{\delta_a}'(N_\beta' - N_{\delta_r}'Y_v/Y_{\delta_r}^*)/N_{\delta_a}' - (L_\beta' - L_{\delta_r}'Y_v/Y_{\delta_r}^*)\}/L_{\delta_r}']_m$

[a] Notice that $V_{T_0}Y_{\delta_r}^* = Y_{\delta_r}$.

The braces on the RHS of Eqs. (32) and (33) are to call attention to the fact that these signals are generated in the filters for obtaining $\ddot{\varphi}^*$ from $\dot{\varphi}$ and a_c from a_y'', respectively. In the latter case, a very modest simplification was afforded by choosing the time constant of the pseudo-differentiator equal to that for the feedback compensation pole.

If it is necessary to include sensor dynamic effects or additional signal conditioning filters, these may be included in $F(s)$ in a manner similar to that for including the flight control servo dynamics. A word of caution is in order, however. The theoretical treatment of system stability given earlier assumes throughout that $F(s) \equiv 1$. Any deviations from this, particularly those which will introduce appreciable lag within the bandwidth of the augmented vehicle, might cause instability when the adaptive loop gains, $A_{\dot{\varphi}}$ and A_y, are at very high values.

The adaptive system mechanization is shown in Fig. 3. It is worthwhile to remark that the amount of equipment required to mechanize this adaptive system is indeed modest. Consider that equipment comparable in complexity to that spanned by arrow A in Fig. 3 is required for any system in which gains are adjusted, as for example, in an air-data scheduled system. The equipment spanned by arrow B in Fig. 3 is specificially for the adaptive control function mechanization. Required are two electronic multiplications, three second order filters (two with real poles), and one first-order filter. These relatively modest equipment requirements for the adaptive control function mechanization are remarkable

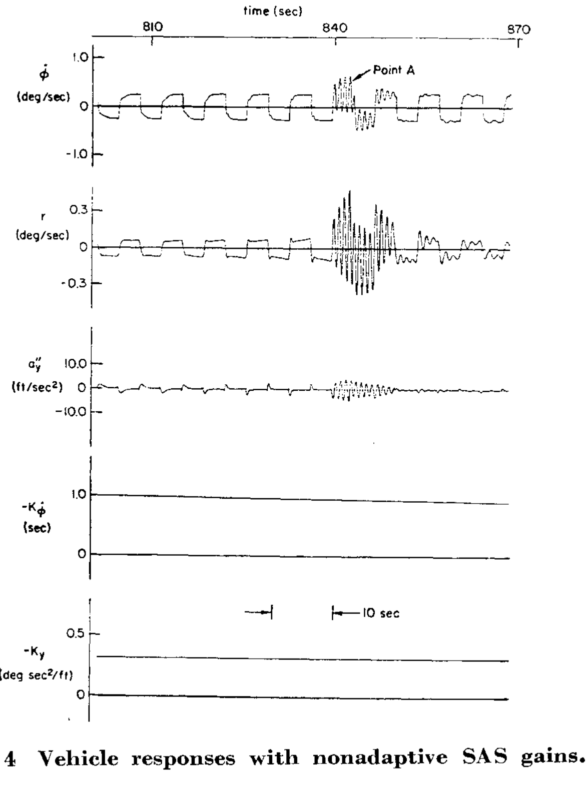

Fig. 4 Vehicle responses with nonadaptive SAS gains.

considering the breadth of capabilities for which the system is designed, and in contrast to the amount of equipment required for some earlier adaptive system designs. See for example, Ref. 2, and 6–10, each of which must be regarded as a competent and successful effort.

Simulation Results

The MLEV equations of motion are assumed to have constant coefficients in the preceding analysis. In actual fact, the differential equations describing the vehicles are time-varying. Although the time-varying effects are negligible for engineering purposes over most of the given flight profile, this is not the case for the 10 sec or so following flight condition 840 during which the vehicle is in the transonic flight regime. Simulation enables evaluation of adaptive system performance in this critical regime. It also is used to illustrate the tolerance of the system to mismatch between the MLEV model and the actual MLEV, to disturbance inputs, to the presence of outer loops, and to the cut-off filtering employed for practical reasons. The implied closed-loop system model is given by Eqs. (9) and (10) [with m subscripts added]. The coefficient values used in the model are those given in Appendix A of Ref. 3 for flight condition 810 except for Y_v. The Y_{v_m} used is ten times the value for flight condition 810. This is to increase the damping ratio of the poles of the filter in Fig. 2.

Time-varying coefficients and trim conditions for the MLEV are given in Appendix C of Ref. 3. The MLEV dynamics are such that no single constant gain setting in the lateral acceleration-to-rudder loop could produce a stable system at every flight condition considered. It was found to be true, however, that when the time-varying vehicle was simulated, a constant value of this gain could be found such that any unstable conditions were sufficiently short-lived that they would not necessarily be disastrous. Simulation data in Fig. 4 illustrate this point. Shown are the roll rate yaw rate, and lateral acceleration (at the sensor) responses of the time-varying system to a series of step roll rate commands with the adaptive system inoperative. The "instability" at point A is the result of a large, rapid change in rudder rolling moment control effectiveness as the vehicle

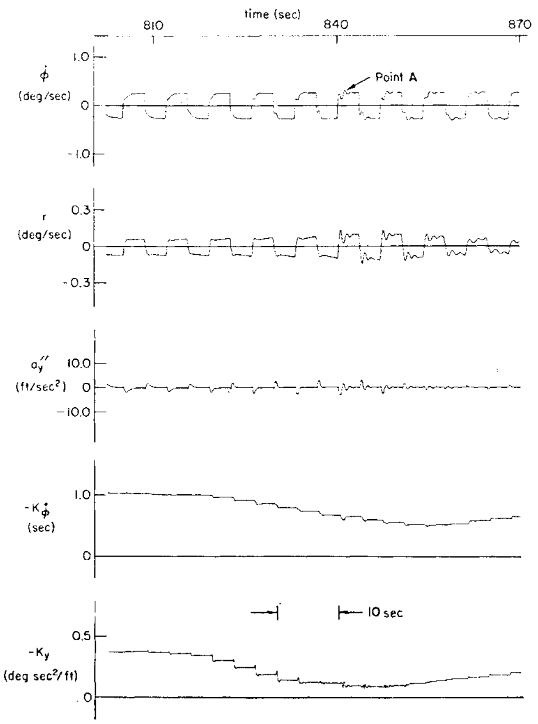

Fig. 5 Vehicle responses with adaptive SAS gains.

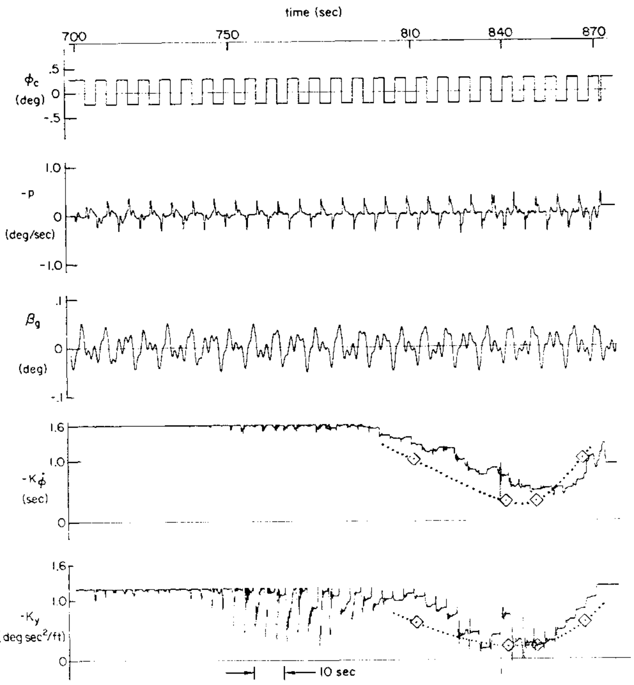

Fig. 6 Roll rate and adaptive gain responses for φ_c and β_g inputs, and comparison of optimum (.......) and adaptively selected (————) gains.

passes through the transonic regime. A very similar problem can result from large changes in angle of attack such as attend the landing flare maneuvers of lifting-body vehicles. Figure 5 shows the same responses as Fig. 4 plus the time histories of the roll rate and lateral acceleration loop gains. Conditions for this set of data are identical to those for the first set of responses except that the adaptive system is operative. The adaptive system now adjusts the loop gains so there is no hint of the instability where it formerly occurred, and it renders all the responses, at all times, more uniform and acceptable.

Figure 6 demonstrates system tolerances to mismatch, disturbance inputs, and outer loop closures. For this figure, a model of the pilot loop closure, φ to $\dot{\varphi}_c$, was included in the simulation. A square wave with zero mean and seven second period was used as a roll angle command. In addition, a pseudo-random side gust disturbed the vehicle. The data intervals are longer so that conditions of increased mismatch are included. Upper limits of $(-K_{\dot{\varphi}})_{\max} = 1.6$ sec and of $(-K_y)_{\max} = 0.6°$ sec^2/ft are placed on the adaptive gains to restrain them from unrealistically compensating for lack of control effectiveness at the low dynamic pressure flight condition.

Comparison of Figs. 5 and 6 shows clearly that the presence of the gust disturbance inputs causes the $(-K_{\dot{\varphi}})$ and $(-K_y)$ adaptive gains to modulate the servo errors $(\dot{\varphi}_c - \dot{\varphi})$ and $(a_{c_c} - a_c)$ more extensively. This, of course, is as it should be in order that the adaptive system produces control functions δ_a and δ_r, which will tend to suppress the disturbance effects on vehicle motion. Disturbance inputs also tend to produce higher average values of the adaptive gains. However, even in the absence of command inputs, these higher average values never caused instability. This was found to be the case for several increased values of the adaptive loop gains $A_{\dot{\varphi}}$ and A_y, which ranged over two orders of magnitude.

A comparison of the loop gains set by the adaptive system with those selected by a competent aircraft flight control system analyst as optimum also is made in Fig. 6. The agreement turns out to be remarkable, and confirms in another way that this adaptive control function system performs in a very reasonable fashion.

Conclusions

The important conclusions are as follows. The adaptive control function technique can be applied to design adaptive multipoint controllers that can be proven stable under certain ideal conditions. These systems are remarkably simple to mechanize and can incorporate such practical modifications as filters for signal conditioning and/or noise suppression. Application of this technique to a practical lateral flight control problem demonstrated its effectiveness, performance, and its practical advantages.

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Quasi-Optimum Design of an Aircraft Landing Control System

Chong K. Ling*

Singer-General Precision Inc., Little Falls, N. J.

An explicit quasi-optimum control law for longitudinal aircraft motion with particular application to the landing phase is obtained by use of Friedland's[1,2] quasi-optimum control technique. A worst-case simulation study (in which the initial errors in altitude, pitch, and angle of attack are −100 ft, −5.4°, and −2.2°, respectively) was performed for a typical jet transport. The resulting trajectory indicated that the aircraft is returned to the desired nominal trajectory in about 20 sec and does not deviate from the nominal trajectory by more than −130 ft. Simulated performance in the presence of steady wind disturbances was also determined to be satisfactory.

1. Introduction

DESPITE the rapid development in the modern optimal feedback control theory and its various applications in the control of space-oriented systems, its potential application to the control of aircraft appears relatively unexplored. The main reason for the lack of enthusiasm may be attributed in part to the fact that any realistic formulations of aircraft control problems unavoidably result in complicated mathematical expressions which are difficult to handle.

In the present paper, attention is focused on the control of aircraft elevator during the final phases of landing. The problem of aircraft landing control with quadratic performance criteria was previously reported by Ellert and Merriam,[3] who propose to guide a landing aircraft to a certain prescribed ideal track at the end of a prespecified period of time. Various performance requirements are then satisfied by a careful adjustment of coefficients in a quadratic performance integral. In this paper, the problem is formulated and approached from a somewhat different point of view. With the designation of an ideal landing trajectory y^*, a nominal elevator deflection δ^* is defined to guide the aircraft to fly along the prespecified path when no deviation therefrom is present. The nominal control is a fixed function of the nominal trajectory and, therefore, is a fixed function of the aircraft position in space. Any deviation of the aircraft dynamic variables from the values defined by the nominal trajectories is regarded as path errors. The control δ, in the presence of path errors is necessarily different from that of the nominal control and the difference $\Delta\delta = \delta - \delta^*$ is to be designed to eliminate the path errors in a manner specified by the performance index. The design of the error correction control is accomplished by applying the quasi-optimum control technique developed by Friedland,[1,2] which results in a feedback control law. Thus, if the aircraft enters the final phase of landing with its altitude, pitch angle, pitch rate and angle of attack different from specified values, the control system is to eliminate these deviations and to maintain the deviation-free condition thereafter. Since the control law is in the form of a feedback configuration, deviations due to outside disturbance are also corrected.

One convenience the quasi-optimum control technique offers is that nonlinearities can be handled effectively to allow realistic formulation of the problem. Moreover, application of the technique yields a feedback control law in closed form that can be readily mechanized.

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* Staff Scientist, Research Center, Kearfott Division.